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ANALYSIS OF OBSERVATIONS OF EARLIEST VISIBILITY OF THE LUNAR CRESCENT

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Predicting the visibility of thin lunar-crescents following the new moon is difficult and challenging for several technical reasons. The visibility of the earliest new moon has long been used to determine the lunar-crescent calendar and is still used today. Many criteria exist for the first visibility of the lunar crescent. Here, we test the most-commonly-used criteria for thin-lunar-crescent visibility. We used 545 observations, including both positive and negative sightings, made by professional and highly-trained astronomers over a duration of 27 years (1988 – 2015) and from different locations at latitudes between 20°N and 29°N (within Saudi Arabia). We developed a new criterion for lunar-crescent visibility using lunar-crescent width (*W*) and the arc of vision (ARCV). This new model can be used to predict the visibility of the lunar crescent by naked eye or aided eye, which is fundamental for the lunar-crescent calendar followed by several cultures and religions.

Introduction

Prediction of thin-lunar-crescent visibility following the new moon is fundamental for the lunar-crescent calendar followed by several cultures and religions (*e.g.*, Muslim, Hindu, and Hebrew). Determination of the new lunar Hijri month (Muslim calendar) is based on the visibility of the earliest, thin, new lunar crescent after sunset. Due to the lack of accurate criteria for thin-lunar-crescent visibility, people scan the western horizon at sunset hoping to sight the lunar crescent, which might be impossible. In the absence of positive lunar-crescent sightings, the beginning of the next lunar month is postponed by one day. Therefore, the lunar calendar cannot be based on observations because it is necessary to wait approximately 29 days to determine the start of the next month. This type of observational calendar is highly affected by atmospheric conditions and typically involves error due to human factors such as illusion[1,2].

The problem of lunar-crescent visibility is very old. The Babylonians established criteria for the first visibility of the lunar crescent based on observational data, which states that a thin lunar crescent can be sighted only if the lunar-crescent altitude above the horizon is greater than 12° after sunset[3]. During the 8th to 14th Centuries, several famous Muslim astronomers developed first-visibility criteria for the thin, new lunar crescent after sunset. One criterion stated that the crescent can be observed only if the arc of separation of the Sun and Moon along the celestial equator is larger than 12°. Al-Khwārizmī used this condition to construct tables for thin-lunar-crescent visibility[4]. Another criterion, used by Tabarī, states that the new lunar crescent will not be seen if the Sun depression is less than 9°·5 at moonset[5]. These two criteria are based on one variable, but Al-Battānī used several variables[6] because, when using one condition, the crescent will not be sighted according to one arc but to many arcs. Al-Battānī included the azimuth of the Moon relative to the Sun, the Earth–Moon distance, and the width of the lunar crescent as conditions for thin-lunar-crescent visibility. After being developed by Al-Battānī and others, the Babylonian criteria became common usage and no further development in lunar-crescent visibility occurred until the second half of the 19th Century[3].

New lunar-crescent-visibility criteria were introduced by Fotheringham[7], who collected 76 naked-eye observations of the visibility or non-visibility of the crescent. Fotheringham calculated the arc of vision (ARCV) at sunset time and the relative azimuth (DAZ) with respect to the Sun, as shown in Fig. 1. He found that the formula of the curve fitted the second-degree polynomial based on non-visible observations[7]. The ARCV–DAZ curve introduced by Fotheringham has been used in several similar studies. In 1911, Maunder added a further 48 observations and showed that the dividing curve should be based on visible observations and not non-visible observations[8]. He made a new lunar-crescent criterion using a second-degree polynomial to fit a new dividing (ARCV–DAZ) curve[8]. The Maunder criterion is similar to the so-called Indian criterion, published in the *Indian Astronomical Ephemeris*[9]. In 1977, Bruin published a new approach addressing the earliest-lunar-crescent visibility[6], using new observations on factors such as lunar-crescent width (W), solar depression, and the necessary contrast for naked-eye observations of the lunar crescent. The criterion adopted a minimum lunar-crescent width ($W = 0'\cdot5$), which represents the limit of lunar-crescent visibility[6]. In 1981, Ilyas estimated that Bruin's visibility threshold was an overestimate, and calculated the minimum lunar width as $W = 0'\cdot25$, subsequently determining a new approach based on dividing (ARCV–DAZ) curve, which shows good agreement with Maunder's criterion[10].

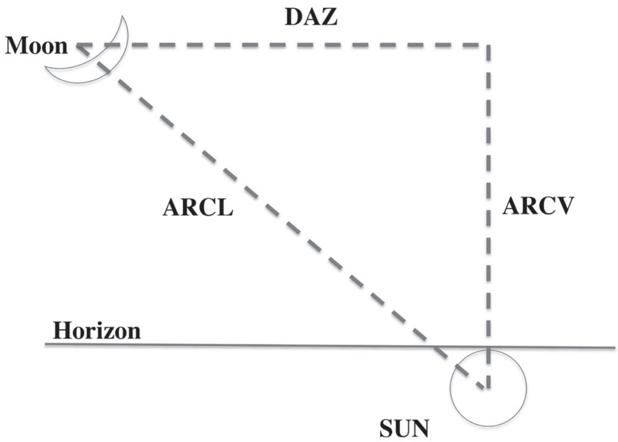

FIG. 1
Schematic showing the position of the Moon after sunset with various celestial arcs.

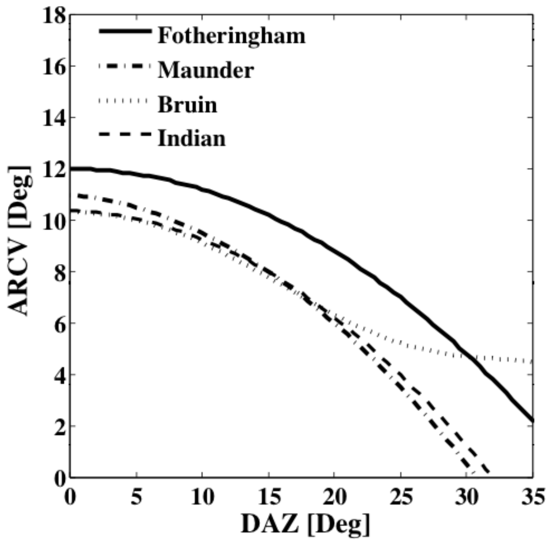

FIG. 2
Fotheringham, Maunder, Bruin, and Indian curves of ARCV = f(DAZ).

In Fig. 2, the geometry-based criteria mentioned above have been designed based on the arc of vision (ARCV) and the relative azimuth (DAZ). A hypothetical line, ARCV = *f*(DAZ), separates the plane into two regions; below the curve represents the invisible lunar crescent and above the curve is the visible lunar crescent. Another important lunar-crescent-visibility condition, recognized as one of the most reliable criteria, is the Danjon limit[11], which states that the lunar crescent disappears and is not visible when the arc of light (ARCL) is less than 7°. In 1998, Yallop[12] improved the Indian criterion using 295 observations over the period 1859 to 1996, and used the topocentric instead of geocentric lunar-crescent width (W). In this study, we test all the criteria for earliest lunar-crescent visibility discussed above by applying them to observations made by highly trained lunar-crescent observers. We also examine the individual conditions such as Moon age, Moon time lag, and the Danjon limit. Then, we propose an improved lunar-crescent criterion based on the (ARCV–DAZ) visibility curve for predicting thin lunar-crescent visibility following the new moon that is suitable for the lunar-crescent calendar.

Observation collection

Observations of the earliest lunar crescent were performed every month from 1988 to 2015. These observations were only made by astronomers at the National Center for Astronomy, who are capable and highly trained to find the earliest lunar crescent following the new moon. The earliest lunar sightings were made after extensive preparation and using a computerized mounted telescope. The position of the Moon was accurately determined with respect to the sunset position. The observations were made in several different locations and all observation reports were written immediately. This data set is fundamental for this study and consists of 288 positive sightings of earliest lunar crescents presented in Table I, and 190 negative sightings in Table II, both made in a clear sky. Fig. 3 shows the percentage of negative and positive sightings and the corresponding observation methods. Because the visibility of the earliest lunar crescent is strongly related to the atmospheric conditions, only observations in clear-sky conditions are considered. For each observation, we calculate the most common observation parameters used to predict the visibility of the earliest lunar crescent. These include the arc of vision (ARCV), which is the Moon's altitude separation from the Sun, the arc of light (ARCL), which is the ecliptic-longitude separation between Moon and Sun, or elongation, and the relative azimuth (DAZ), which is the azimuthal separation of the Moon and Sun. These arcs are illustrated in Fig. 1.

As well as these arcs, we calculated the Moon's age or elapsed time between the new moon conjunction and sunset on the day of observation, the Moon's lag time (LAG), or the elapsed time between sunset and moonset on the day of observation, and the lunar-crescent width, (W), which is the illuminated Moon's surface measured along the Moon's diameter ($W = 0'$ at new moon and $32'$ at full moon). Examples of observations and observation parameters for the early lunar crescent are given in Table III. The ARCV and ARCL distributions for the 288 positive sightings are shown in Figs. 4 and 5, respectively. The solid curves in these figures represent the best Gaussian fits of the distributions. The mean Gaussian fits for the ARCV and ARCL were 13°·31 and 16°·21, respectively, and the standard deviations were 3°·50 and 4°·02. Only four false positive sightings were eliminated from our data collection, which varied from the observations by three standard deviations[13,14].

Table I

Positive observations of earliest lunar crescent

T stands for telescope; and N stands for naked-eye.

Lat. °	*Long.* °	*Date UT*	*Age hrs.*	*Lag min.*	*ARCV* °	*ARCL* °	*DAZ* °	*Means*
23·55	46·39	1988/03/19	37·05	93	20·34	20·49	02·55	N
19·52	42·22	1988/04/17	27·49	65	14·15	14·83	04·49	T
23·56	46·93	1988/04/17	27·28	68	14·34	14·70	03·27	T
28·40	36·73	1989/03/08	21·31	56	11·90	11·97	01·11	T
28·40	36·73	1990/02/26	30·61	77	16·36	16·41	01·33	N
26·45	36·38	1990/03/27	20·01	51	10·99	11·67	03·95	T
28·40	36·73	1990/03/27	20·00	53	11·11	11·65	03·54	N
28·40	36·73	1990/04/26	35·62	107	20·88	20·91	01·12	N
26·45	36·38	1990/06/23	21·60	55	10·94	11·84	04·54	T
28·40	36·73	1991/04/15	20·35	54	11·02	11·53	03·41	T
26·45	36·38	1991/05/15	35·64	101	19·98	20·11	02·27	N
28·40	36·73	1992/03/05	26·02	56	11·90	12·49	03·82	N
28·40	36·73	1992/06/02	36·01	89	17·62	19·57	08·65	N
26·45	36·38	1992/07/01	20·94	55	11·42	15·90	01·14	N
21·42	39·83	1993/12/14	29·23	72	14·71	15·31	04·28	N
28·40	36·73	1993/01/23	20·64	46	09·45	09·53	01·23	T
28·40	36·73	1993/02/22	26·39	55	11·62	12·12	03·46	T
21·42	39·83	1993/12/14	29·23	72	14·71	15·31	04·28	N
24·60	46·45	1998/10/21	28·22	53	11·46	12·86	05·87	N
27·52	41·70	1998/12/20	40·20	89	16·95	18·35	07·13	N
26·35	43·95	1999/01/18	22·83	52	10·44	10·65	02·13	T
24·60	46·50	1999/03/18	20·27	48	10·54	11·30	04·11	T
26·35	43·95	1999/10/10	27·12	53	11·41	13·15	06·58	N
24·60	46·45	2000/03/07	34·01	74	16·12	17·00	05·26	N
21·42	39·83	2000/09/28	19·30	44	10·10	10·77	03·75	T
27·52	41·70	2000/12/26	21·08	40	07·69	08·86	04·40	T
21·42	39·83	2001/01/25	25·99	49	10·31	11·23	04·46	T
24·60	46·45	2001/01/25	25·46	48	09·80	11·02	05·05	T
28·40	36·73	2001/02/24	31·14	62	12·77	14·31	06·53	N
28·40	36·73	2001/04/24	24·64	52	10·81	12·42	06·14	N
21·42	39·83	2001/04/24	24·29	51	11·31	12·21	04·62	N
21·42	39·83	2001/06/22	28·13	67	14·09	15·21	05·78	N
27·52	41·70	2001/11/16	31·68	64	12·39	16·21	10·53	N
24·60	46·45	2002/01/14	24·96	48	09·53	11·51	06·49	N
27·52	41·70	2002/02/13	31·37	61	12·52	14·34	07·06	N
26·35	43·95	2002/10/07	27·45	57	12·26	16·00	10·36	N
27·52	41·70	2002/11/05	17·88	36	07·41	09·90	06·58	T
26·35	43·95	2003/02/02	28·16	59	11·85	14·08	07·66	N
27·52	41·70	2003/02/02	28·13	59	11·74	14·14	07·94	N
21·42	39·83	2003/02/02	28·17	60	12·86	14·22	06·50	N
28·40	36·70	2003/02/02	28·16	60	11·76	14·29	08·19	N
24·60	46·45	2003/02/02	28·13	59	12·03	14·01	07·23	N
28·40	36·73	2003/08/28	22·55	49	10·52	12·36	06·52	T
27·52	41·70	2003/09/27	35·93	65	13·79	20·04	14·68	N
21·42	39·83	2003/09/27	36·06	68	15·30	20·30	13·12	N
26·35	43·95	2003/09/27	35·78	65	14·04	19·94	14·31	N
28·40	36·73	2003/09/27	36·26	65	13·67	20·23	15·07	N
27·52	41·70	2003/10/26	25·74	46	09·39	14·55	11·17	T
28·40	36·73	2003/12/24	28·99	65	11·48	16·86	12·43	N
21·42	39·83	2003/12/24	29·03	68	13·05	16·81	10·68	N
26·35	43·95	2003/12/24	28·59	65	11·76	16·61	11·81	N
24·60	46·45	2004/01/23	41·46	103	20·37	22·79	10·45	N
27·52	41·70	2004/01/23	41·69	104	19·89	22·93	11·66	N
26·35	43·95	2004/01/23	41·58	103	20·08	22·87	11·18	N
28·40	36·73	2004/01/23	42·00	105	19·85	23·10	12·06	N
21·42	39·83	2004/05/20	35·04	75	15·35	15·35	00·08	N
24·54	39·63	2004/06/18	19·76	46	09·07	9·07	00·16	T
21·42	39·83	2004/06/19	43·64	61	12·25	19·56	04·11	N

TABLE I (*continued*)

Lat. °	*Long.* °	*Date* *UT*	*Age* *hrs.*	*Lag* *min.*	*ARCV* °	*ARCL* °	*DAZ* °	*Means*
24·60	46·45	2004/06/19	43·31	95	18·86	19·42	04·73	N
27·52	41·70	2004/08/17	38·42	67	14·49	18·38	11·43	N
28·40	36·73	2004/09/15	25·15	41	08·91	9·05	01·56	T
21·42	39·83	2004/09/16	48·92	78	17·63	21·03	11·66	N
28·40	36·73	2005/01/11	26·88	67	12·19	16·11	10·61	N
24·54	39·63	2005/02/09	16·74	40	8·39	09·88	05·22	T
21·42	39·83	2005/08/06	21·19	64	14·30	16·36	08·03	N
24·60	46·45	2005/08/06	20·83	63	13·69	16·22	08·79	N
26·35	43·95	2005/08/06	21·04	63	13·46	16·33	09·33	N
24·54	39·63	2005/09/05	44·34	61	13·76	13·77	0·40	N
26·35	43·95	2005/11/03	37·33	51	09·65	18·90	16·33	N
27·52	41·70	2005/11/03	37·45	50	09·34	18·98	16·59	N
21·42	39·83	2006/01/01	48·59	89	17·09	20·33	11·19	N
24·54	39·63	2006/01/01	48·50	88	16·34	20·31	12·23	N
24·54	39·63	2006/02/28	14·88	36	07·91	08·11	01·81	T
28·40	36·73	2006/07/26	35·94	64	13·33	16·44	09·70	N
26·35	43·95	2006/12/21	24·27	43	07·83	12·81	10·17	T
27·52	41·70	2006/12/22	48·38	108	18·71	24·82	16·62	N
21·42	39·83	2007/08/14	40·83	60	13·81	19·35	13·69	N
28·40	36·73	2008/08/02	29·81	47	10·02	16·07	12·63	N
27·52	41·70	2008/12/29	50·43	102	19·00	21·89	11·09	N
24·54	39·63	2009/01/27	31·14	65	13·56	13·64	01·53	N
26·35	43·95	2009/01/27	30·80	65	13·34	13·49	02·00	N
27·52	41·70	2009/01/27	30·92	66	13·35	13·55	02·32	N
27·52	41·70	2009/04/26	36·36	101	19·88	19·93	01·41	N
24·60	46·45	2009/07/23	37·14	66	14·62	21·36	15·74	N
26·35	43·95	2009/07/23	37·36	65	14·18	21·51	16·34	N
24·60	46·45	2009/08/21	29·36	36	08·18	14·67	12·22	T
26·35	43·95	2009/10/19	32·99	35	06·99	17·47	16·05	T
26·35	43·95	2009/11/18	42·85	70	12·74	20·57	16·28	N
25·62	45·62	2009/11/18	42·88	70	12·92	20·52	16·07	N
25·62	45·62	2009/12/17	26·13	48	09·09	11·50	07·07	T
25·25	45·25	2009/12/17	26·17	48	09·16	11·51	07·01	T
28·40	36·73	2010/04/15	27·49	65	13·21	13·51	02·85	N
24·54	39·63	2010/04/15	27·24	61	12·85	13·40	03·84	N
24·54	39·63	2010/06/13	28·94	68	13·87	15·22	06·34	N
26·35	43·95	2010/06/13	28·72	67	13·53	15·11	06·79	N
28·40	36·73	2010/06/13	29·28	68	13·50	15·44	07·55	N
25·25	45·25	2010/06/13	28·59	67	13·61	15·03	06·44	N
28·40	36·73	2010/08/11	37·13	48	10·37	18·98	15·99	T
26·37	49·82	2010/08/11	36·21	48	10·74	19·30	16·13	N
24·39	39·63	2010/10/09	44·27	65	13·08	25·29	21·83	N
24·60	46·45	2010/10/09	43·82	63	12·83	25·04	21·69	N
26·35	43·95	2010/10/09	43·97	61	12·23	25·15	22·14	N
25·25	45·25	2010/10/09	43·89	63	12·62	25·09	21·87	N
26·37	49·82	2010/10/09	43·58	61	12·08	24·93	21·98	N
25·62	45·62	2010/10/09	43·86	62	12·47	25·08	21·94	N
27·52	41·70	2010/11/07	33·57	53	10·06	18·09	15·11	T
21·42	39·83	2010/11/07	33·83	59	11·77	18·15	13·92	N
24·39	39·63	2010/11/07	33·78	56	10·99	18·16	14·55	N
24·60	46·45	2010/11/07	33·32	55	10·74	17·92	14·44	N
26·35	43·95	2010/11/07	33·44	54	10·32	18·01	14·84	T
25·25	45·25	2010/11/07	33·38	54	10·59	17·97	14·59	T
24·39	39·63	2010/12/06	20·51	40	07·87	10·20	06·50	T
24·60	46·45	2010/12/06	20·05	39	07·64	09·97	06·42	T
25·62	45·62	2010/12/06	20·07	39	07·52	09·99	06·59	T
24·39	39·63	2011/01/05	29·28	67	13·50	13·75	02·67	N
24·60	46·45	2011/01/05	29·82	66	13·27	13·54	02·68	N
24·39	39·63	2011/04/04	24·80	50	10·70	11·43	04·04	T
21·42	39·83	2011/05/04	32·74	70	14·74	14·76	00·79	N
24·39	39·63	2011/05/04	32·82	72	14·80	14·80	00·09	N

TABLE I (*continued*)

Lat. °	*Long.* °	*Date UT*	*Age hrs.*	*Lag min.*	*ARCV* °	*ARCL* °	*DAZ* °	*Means*
27·52	41·70	2011/05/04	32·76	74	14·74	14·77	01·00	N
21·42	39·83	2011/07/02	31·46	56	12·10	15·96	10·48	N
24·39	39·63	2011/07/02	31·57	54	11·53	16·05	11·24	N
24·60	46·45	2011/07/02	31·12	53	11·30	15·82	11·14	N
28·40	36·73	2011/07/02	31·91	52	10·76	16·28	12·29	T
25·62	45·62	2011/07/02	31·22	53	11·12	15·88	11·41	N
26·37	49·82	2011/08/30	35·99	41	09·07	09·22	01·70	T
24·60	46·45	2011/09/28	27·60	32	06·98	16·55	15·04	T
21·42	39·83	2011/10/28	48·86	84	16·90	24·63	18·19	N
24·39	39·63	2011/10/28	48·82	82	15·90	24·64	19·07	N
24·60	46·45	2011/10/28	48·36	80	15·63	24·38	18·95	N
28·40	36·73	2011/10/28	48·94	78	14·56	24·75	20·24	N
25·25	45·25	2011/10/28	48·43	80	15·44	24·42	19·16	N
25·62	45·62	2011/10/28	48·40	79	15·30	24·41	19·25	N
26·37	49·82	2011/10/28	48·10	78	14·93	24·25	19·34	N
24·60	46·45	2011/11/26	31·92	73	4·12	17·56	10·55	N
28·40	36·73	2011/11/26	32·45	74	13·57	17·90	11·78	N
25·25	45·25	2011/11/26	31·98	73	14·02	17·60	10·75	N
25·62	45·62	2011/11/26	31·95	73	13·93	17·59	10·84	N
24·39	39·63	2011/12/25	20·57	52	10·49	10·73	02·29	T
24·60	46·45	2011/12/25	20·10	51	10·24	10·48	02·25	T
26·35	43·95	2011/12/25	20·21	52	10·21	10·55	02·65	T
28·40	36·73	2011/12/25	20·61	54	10·30	10·77	03·18	T
25·25	45·25	2011/12/25	20·16	51	10·24	10·52	02·40	T
25·62	45·62	2011/12/25	20·12	51	10·21	10·50	02·48	T
21·42	39·83	2012/01/24	31·43	72	15·80	15·97	02·33	N
24·60	46·45	2012/01/24	30·90	74	15·64	15·70	01·36	N
25·25	45·25	2012/01/24	30·96	74	15·69	15·73	01·15	N
25·62	45·62	2012/01/24	30·93	75	15·68	15·72	01·04	N
24·39	39·63	2012/04/22	32·47	66	13·90	13·90	00·26	N
21·42	39·83	2012/07/20	35·68	48	10·74	17·87	14·36	N
24·60	46·45	2012/07/20	35·33	44	09·74	17·74	14·90	N
24·39	39·63	2012/07/20	35·78	45	09·96	17·95	15·01	N
26·35	43·95	2012/07/20	35·56	43	09·32	17·87	15·31	T
25·62	45·62	2012/07/20	35·42	43	09·49	17·80	15·13	T
21·42	39·83	2012/09/17	37·18	50	11·25	16·73	12·46	N
24·60	46·45	2012/09/17	36·75	46	10·12	16·17	12·68	N
26·35	43·95	2012/09/17	36·92	45	09·64	16·10	12·95	T
25·25	45·25	2012/09/17	36·83	46	09·95	16·15	12·79	T
26·37	49·82	2012/09/17	36·53	44	09·49	15·86	12·77	T
25·62	45·62	2012/09/17	36·81	45	09·83	16·10	12·82	T
24·60	46·45	2012/10/16	26·40	40	08·49	14·85	12·23	T
26·35	43·95	2012/10/16	26·54	39	08·17	14·95	12·57	T
25·25	45·25	2012/10/16	26·47	40	08·38	14·90	12·36	T
25·62	45·62	2012/10/16	26·44	40	08·28	14·89	12·41	T
21·42	39·83	2012/11/15	39·51	97	19·33	23·26	13·19	N
24·39	39·63	2012/11/15	39·45	96	18·55	23·25	14·27	N
24·60	46·45	2012/11/15	38·99	95	18·28	22·98	14·18	N
26·35	43·95	2012/11/15	39·11	95	17·87	23·07	14·84	N
27·52	41·70	2012/11/15	39·22	95	17·60	23·15	15·29	N
28·40	36·73	2012/11/15	39·53	95	17·48	23·34	15·71	N
25·25	45·25	2012/11/15	39·05	95	18·14	23·03	14·43	N
25·62	45·62	2012/11/15	39·02	95	18·03	23·01	14·55	N
26·37	49·82	2012/11/15	38·71	93	17·68	22·84	14·70	N
21·42	39·83	2012/12/14	29·99	82	16·65	17·23	04·51	N
24·39	39·63	2012/12/14	29·90	82	16·31	17·19	05·50	N
24·60	46·45	2012/12/14	29·44	81	16·05	16·92	05·46	N
26·35	43·95	2012/12/14	29·54	82	15·90	16·99	06·06	N
27·52	41·70	2012/12/14	29·65	83	15·82	17·07	06·48	N
25·25	45·25	2012/12/14	29·50	82	16·00	16·96	05·69	N
25·62	45·62	2012/12/14	29·46	82	15·94	16·94	05·80	N
26·35	43·95	2013/01/12	18·80	53	10·93	10·98	01·07	T

Table I (*continued*)

Lat. °	*Long.* °	*Date* UT	*Age* hrs.	*Lag* min.	*ARCV* °	*ARCL* °	*DAZ* °	*Means*
25·62	45·62	2013/01/12	18·71	53	10·87	10·94	01·25	T
26·37	49·82	2013/01/12	18·41	52	10·71	10·77	01·13	T
21·42	39·83	2013/02/11	31·94	75	16·72	17·20	04·10	N
24·39	39·63	2013/02/11	31·90	77	16·89	17·17	03·13	N
28·40	36·73	2013/02/11	32·01	81	17·13	17·22	01·79	N
25·25	45·25	2013/02/11	31·51	77	16·72	16·96	02·88	N
25·62	45·62	2013/02/11	31·47	77	16·73	16·95	02·76	N
26·37	49·82	2013/02/11	31·18	77	16·61	16·79	02·54	N
21·42	39·83	2013/04/11	30·06	62	13·61	13·65	01·02	N
21·42	39·83	2013/10/06	38·52	61	13·24	19·63	14·63	N
24·39	39·63	2013/10/06	38·48	59	12·46	19·66	15·33	N
26·35	43·95	2013/10/06	38·92	60	11·82	19·52	15·65	N
27·52	41·70	2013/10/06	38·31	56	11·55	19·61	15·95	N
25·25	45·25	2013/10/06	38·40	57	12·09	19·47	15·38	N
25·62	45·62	2013/10/06	38·40	57	11·98	19·46	15·45	N
26·37	49·82	2013/10/06	38·29	56	11·67	19·32	15·50	N
21·42	39·83	2013/11/04	25·88	54	11·25	13·74	07·93	N
24·39	39·63	2013/11/04	25·83	52	10·79	13·74	08·56	T
24·72	46·67	2013/11/04	25·35	51	10·53	13·47	08·46	T
25·25	45·25	2013/11/04	25·35	51	10·48	13·52	08·60	T
24·39	39·63	2013/12/03	14·18	39	07·92	08·10	01·62	T
21·42	39·83	2014/01/02	27·62	78	16·34	16·34	00·22	N
24·39	39·63	2014/01/02	27·50	79	16·25	16·29	01·20	N
24·72	46·67	2014/01/02	27·03	78	15·97	16·02	01·21	N
26·35	43·95	2014/01/02	27·04	80	15·99	16·09	01·78	N
27·52	41·70	2014/01/02	27·58	81	16·01	16·16	02·20	N
28·40	36·73	2014/01/02	27·58	83	16·13	16·33	02·56	N
25·25	45·25	2014/01/02	27·02	79	16·00	16·06	01·41	N
25·62	45·62	2014/01/02	27·02	79	15·97	16·04	01·52	N
26·37	49·82	2014/01/02	27·58	79	15·78	15·87	01·70	N
24·39	39·63	2014/01/31	17·46	47	10·13	10·56	03·01	T
24·72	46·67	2014/01/31	16·99	46	09·87	10·31	03·00	T
26·35	43·95	2014/01/31	17·13	47	10·04	10·38	02·65	T
25·25	45·25	2014/01/31	17·07	46	09·95	10·35	02·88	T
25·62	45·62	2014/01/31	17·03	47	09·95	10·33	02·81	T
21·42	39·83	2014/03/02	31·42	75	16·99	17·23	02·94	N
24·72	46·67	2014/03/02	30·93	77	16·86	16·96	01·90	N
26·35	43·95	2014/03/02	30·94	78	16·99	17·05	01·37	N
25·25	45·25	2014/03/02	30·94	77	16·92	17·01	01·73	N
25·62	45·62	2014/03/02	30·94	78	16·92	16·99	01·61	N
26·37	49·82	2014/03/02	31·14	77	16·78	16·83	01·39	N
21·42	39·83	2014/03/31	20·84	46	10·42	10·46	00·89	T
24·39	39·63	2014/03/31	20·87	48	10·47	10·48	00·30	T
21·42	39·83	2014/04/30	33·52	73	15·91	16·37	03·91	N
24·39	39·63	2014/04/30	33·60	74	15·71	16·42	04·83	N
28·40	36·73	2014/04/30	33·88	75	15·45	16·58	06·10	N
25·25	45·25	2014/04/30	33·24	73	15·46	16·24	05·05	N
25·62	45·62	2014/04/30	33·23	73	15·41	16·23	05·16	N
21·42	39·83	2014/05/29	21·30	38	08·11	10·37	06·48	T
24·72	46·67	2014/05/29	20·95	36	07·54	10·25	06·97	T
27·52	41·70	2014/05/29	21·37	36	07·33	10·47	07·50	T
25·62	45·62	2014/05/29	21·05	36	07·46	10·30	07·13	T
26·37	49·82	2014/05/29	20·79	35	07·24	10·20	07·20	T
26·35	43·95	2014/07/28	41·35	46	09·94	18·71	15·93	T
21·42	39·83	2014/09/25	33·01	44	10·01	14·90	11·10	N
24·72	46·67	2014/09/25	32·55	42	09·20	14·72	11·55	T
26·35	43·95	2014/09/25	32·73	41	08·93	14·83	11·89	T
27·52	41·70	2014/09/25	32·88	41	08·73	14·91	12·13	T
28·40	36·73	2014/09/25	33·21	41	08·66	15·08	12·39	T
25·62	45·62	2014/09/25	32·62	41	09·04	14·77	11·72	N
26·35	43·95	2014/11/23	25·67	62	12·31	13·41	05·36	N

TABLE I (*concluded*)

Lat. °	*Long.* °	*Date* *UT*	*Age* *hrs.*	*Lag* *min.*	*ARCV* °	*ARCL* °	*DAZ* °	*Means*
27·52	41·70	2014/11/23	25·81	62	12·23	13·47	05·69	N
21·42	39·83	2014/12/23	37·15	98	20·37	20·63	03·35	N
24·72	46·67	2014/12/23	36·58	99	19·83	20·33	04·60	N
26·35	43·95	2014/12/23	36·70	101	19·73	20·40	05·31	N
27·52	41·70	2014/12/23	36·81	101	19·66	20·47	05·82	N
28·40	36·73	2014/12/23	37·11	103	19·70	20·64	06·27	N
25·25	45·25	2014/12/23	36·65	100	19·82	20·37	04·84	N
25·62	45·62	2014/12/23	36·62	100	19·76	20·36	04·98	N
26·37	49·82	2014/12/23	36·31	100	19·53	20·19	05·20	N
21·42	39·83	2015/01/21	25·83	68	14·74	14·83	01·61	N
24·72	46·67	2015/01/21	25·28	69	14·50	14·52	00·72	N
26·35	43·95	2015/01/21	25·41	71	14·59	14·59	00·21	N
27·52	41·70	2015/01/21	25·52	72	14·65	14·65	00·16	N
28·40	36·73	2015/01/21	25·83	73	14·82	14·83	00·48	N
25·25	45·25	2015/01/21	25·36	70	14·55	14·56	00·55	N
25·62	45·62	2015/01/21	25·32	70	14·53	14·54	00·44	N
26·37	49·82	2015/01/21	25·02	70	14·37	14·37	00·28	N
28·40	36·73	2015/02/19	15·65	41	08·73	08·82	01·30	T
21·42	39·83	2015/03/21	29·93	74	16·70	16·70	00·16	N
27·52	41·70	2015/03/21	29·81	77	16·51	16·52	00·57	N
28·40	36·73	2015/03/21	30·14	79	16·67	16·69	00·83	N
21·42	39·83	2015/05/19	35·67	79	17·11	19·05	08·51	N
24·39	39·63	2015/05/19	35·77	79	16·66	19·12	09·53	N
24·72	46·67	2015/05/19	35·31	77	16·37	18·89	09·56	N
26·35	43·95	2015/05/19	35·54	78	16·18	19·02	10·14	N
25·25	45·25	2015/05/19	35·42	78	16·33	18·95	09·75	N
25·62	45·62	2015/05/19	35·41	77	16·25	18·94	09·87	N
26·37	49·82	2015/05/19	35·15	77	15·99	18·82	10·07	N
24·39	39·63	2015/06/17	26·11	42	08·99	13·72	10·41	T
26·35	43·95	2015/06/17	25·89	41	08·49	13·64	10·72	T
27·52	41·70	2015/06/17	26·09	40	08·32	13·75	10·98	T
26·37	49·82	2015/06·/17	25·50	39	8·31	13·46	10·63	T
21·42	39·83	2015/09/14	32·73	44	09·95	10·19	02·23	T
26·37	49·82	2015/09/14	32·09	40	08·83	09·16	02·43	T
21·42	39·83	2015/11/13	44·87	91	18·97	21·05	09·31	N
24·72	46·67	2015/11/13	44·33	90	18·16	20·82	10·35	N
26·35	43·95	2015/11/13	44·47	91	17·89	20·90	10·97	N
27·52	41·70	2015/11/13	44·59	90	17·70	20·96	11·42	N
28·40	36·73	2015/11/13	44·90	91	17·62	21·11	11·83	N
25·62	45·62	2015/11/13	44·38	90	18·01	20·85	10·68	N
26·37	49·82	2015/11/13	44·08	90	17·75	20·71	10·85	N
26·37	49·82	2015/11/13	44·08	90	17·75	20·71	10·85	N

TABLE II

Negative observations of earliest lunar crescent (all means)

Lat. °	*Long.* °	*Date* *UT*	*Age* *hrs.*	*Lag* *min.*	*ARCV* °	*ARCL* °	*DAZ* °
23·56	46·39	1988/03/18	13·04	30	06·70	06·71	00·35
19·15	42·22	1988/04/16	03·49	04	01·01	03·28	03·12
23·56	46·39	1988/04/16	03·27	05	01·14	03·17	02·96
28·40	36·73	1990/02/25	06·60	14	03·01	03·45	01·69
28·40	36·73	1990/04/25	11·61	34	06·83	07·77	03·73
26·45	36·38	1991/05/14	11·63	33	06·64	06·99	02·20
28·40	36·73	1992/06/01	04·13	30	05·95	06·18	01·71
21·42	39·83	1993/12/13	05·22	12	02·59	02·63	00·45
28·40	36·73	1993/02/21	02·38	04	00·87	04·71	04·63

TABLE II (*continued*)

Lat. °	*Long.* °	*Date* *UT*	*Age* *hrs.*	*Lag* *min.*	*ARCV* °	*ARCL* °	*DAZ* °
21·42	39·83	1993/12/13	05·22	12	02·59	02·63	00·45
28·40	36·73	1993/02/21	02·38	04	00·87	04·71	04·63
21·42	39·83	1993/12/13	05·22	12	02·59	02·63	00·45
24·60	46·45	1998/10/20	04·23	16	03·60	03·95	01·62
26·35	43·95	1999/10/09	03·14	16	03·66	04·13	01·90
24·60	46·45	2000/03/06	20·79	18	03·92	05·62	04·03
21·42	39·83	2000/09/27	19·32	03	00·71	01·86	01·72
28·40	36·73	2001/02/23	07·13	09	01·88	04·88	04·50
21·42	39·83	2001/06/21	04·13	05	01·15	01·71	01·26
24·60	46·45	2001/10/16	19·06	02	00·64	05·33	05·29
27·52	41·70	2001/11/15	07·69	19	03·97	04·08	00·93
27·52	41·70	2002/02/12	07·36	08	01·80	05·22	04·90
26·35	43·95	2002/10/06	03·47	16	03·65	03·83	01·13
26·35	43·95	2003/02/01	12·20	00	00·22	05·00	05·00
21·42	39·83	2003/02/01	12·59	04	00·86	04·98	04·91
28·40	36·73	2003/02/01	12·62	01	00·24	05·09	05·09
24·60	46·45	2003/02/01	12·07	01	00·32	04·96	04·95
28·40	36·73	2003/08/27	22·57	12	02·51	04·53	03·77
27·52	41·70	2003/09/26	11·95	27	06·08	06·83	03·12
21·42	39·83	2003/09/26	12·08	28	06·43	06·86	02·42
26·35	43·95	2003/09/26	11·80	27	06·10	06·75	02·91
28·40	36·73	2003/09/26	12·28	28	06·14	07·01	03·38
27·52	41·70	2003/10/25	01·76	04	01·04	01·09	00·30
21·42	39·83	2003/12/23	05·02	02	00·51	04·49	04·46
21·42	39·83	2004/05/19	11·03	21	04·48	04·60	01·07
28·40	36·73	2004/09/14	01·17	09	02·15	02·68	01·60
21·42	39·83	2005/08/05	21·20	29	06·33	06·35	00·45
24·60	46·45	2005/08/05	20·84	29	06·18	06·22	00·75
26·35	43·95	2005/08/05	21·05	30	06·21	06·30	01·05
26·35	43·95	2005/11/02	23·34	08	01·72	06·93	06·72
27·52	41·70	2005/11/02	23·47	07	01·62	07·01	06·82
21·42	39·83	2005/12/31	11·63	19	03·88	07·86	06·83
24·54	39·63	2005/12/31	11·54	18	03·42	07·86	07·08
28·40	36·73	2006/07/25	11·95	29	05·89	06·00	01·16
21·42	39·83	2007/08/13	16·84	26	05·94	07·69	04·89
28·40	36·73	2008/08/01	20·42	08	01·91	02·86	02·12
24·54	39·63	2009/01/26	07·12	11	02·39	02·46	00·56
26·35	43·95	2009/01/26	06·79	11	02·22	02·31	00·65
27·52	41·70	2009/01/26	06·90	11	02·26	02·37	00·72
27·52	41·70	2009/04/25	12·35	33	06·67	07·70	03·85
24·60	46·45	2009/07/22	13·15	22	04·78	07·23	05·43
26·35	43·95	2009/07/22	13·37	22	04·68	07·38	05·71
26·35	43·95	2009/11/17	19·00	18	03·61	09·73	09·04
25·62	45·62	2009/11/17	18·91	19	03·70	09·68	08·96
28·40	36·73	2010/04/14	03·49	07	01·49	04·91	04·68
24·54	39·63	2010/04/14	03·24	04	01·04	04·95	04·84
24·54	39·63	2010/06/12	04·93	11	02·28	02·31	00·39
26·35	43·95	2010/06/12	04·71	11	02·19	02·21	00·34
28·40	36·73	2010/06/12	05·27	12	02·45	02·46	00·05
25·25	45·25	2010/06/12	04·58	10	02·12	02·16	00·44
28·40	36·73	2010/08/10	13·15	07	01·74	08·36	08·18
26·37	49·82	2010/08/10	12·23	078	01·69	07·85	07·66
24·39	39·63	2010/10/08	20·29	17	03·69	12·40	11·84
24·60	46·45	2010/10/08	19·83	16	03·47	12·17	11·67
26·35	43·95	2010/10/08	19·99	14	03·17	12·27	11·86
25·25	45·25	2010/10/08	19·91	15	03·37	12·21	11·75
26·37	49·82	2010/10/08	19·60	14	03·02	12·07	11·69
25·62	45·62	2010/10/08	19·88	15	03·28	12·21	11·76
27·52	41·70	2010/11/06	09·58	01	00·29	06·32	06·31
21·42	39·83	2010/11/06	09·84	05	01·11	06·33	06·23
24·39	39·63	2010/11/06	09·79	03	00·74	06·35	06·31

TABLE II (*continued*)

Lat. °	*Long.* °	*Date UT*	*Age hrs.*	*Lag min.*	*ARCV* °	*ARCL* °	*DAZ* °
24·60	46·45	2010/11/06	09·33	02	00·52	06·18	06·16
26·35	43·95	2010/11/06	09·45	01	00·37	06·25	06·24
25·25	45·25	2010/11/06	09·39	02	00·47	06·21	06·20
24·39	39·63	2011/01/04	12·27	11	02·23	02·24	00·23
24·60	46·45	2011/01/04	11·80	09	02·01	02·03	00·28
21·42	39·83	2011/05/03	18·74	17	03·58	04·50	02·74
24·39	39·63	2011/05/03	18·82	18	03·75	04·50	02·49
27·52	41·70	2011/05/03	18·75	19	03·86	04·46	02·23
21·42	39·83	2011/07/01	21·45	07	01·63	03·71	03·34
24·39	39·63	2011/07/01	21·57	07	01·48	03·81	03·51
24·60	46·45	2011/07/01	21·12	06	01·28	03·61	03·38
28·40	36·73	2011/07/01	21·91	06	01·34	04·02	03·80
25·62	45·62	2011/07/01	21·22	06	01·25	03·67	03·45
21·42	39·83	2011/10/27	18·87	27	05·79	11·00	09·37
24·39	39·63	2011/10/27	18·83	25	05·26	11·01	09·69
24·60	46·45	2011/10/27	18·37	24	05·02	10·76	09·53
28·40	36·73	2011/10/27	18·95	23	04·60	11·13	10·15
25·25	45·25	2011/10/27	18·44	24	04·94	10·81	09·63
25·62	45·62	2011/10/27	18·41	23	04·86	10·80	09·65
26·37	49·82	2011/10/27	18·12	22	04·60	10·65	09·61
21·42	39·83	2011/11/25	08·46	13	02·89	04·31	03·20
24·39	39·63	2011/11/25	08·39	13	02·66	04·30	03·38
24·60	46·45	2011/11/25	07·92	11	02·42	04·06	03·26
28·40	36·73	2011/11/25	08·45	12	02·43	04·39	03·66
25·25	45·25	2011/11/25	07·98	11	02·41	04·10	03·32
25·62	45·62	2011/11/25	07·95	11	02·37	04·08	03·33
21·42	39·83	2012/01/23	07·42	16	03·62	05·02	03·49
24·60	46·45	2012/01/23	06·89	16	03·54	04·81	03·26
25·25	45·25	2012/01/23	06·95	17	03·61	04·83	03·20
25·62	45·62	2012/01/23	06·91	17	03·62	04·81	03·17
21·42	39·83	2012/04/21	08·40	14	03·04	03·89	02·43
24·39	39·63	2012/04/21	08·46	15	03·19	03·88	02·21
21·42	39·83	2012/07/19	11·68	06	01·38	07·03	06·90
24·60	46·45	2012/07/19	11·34	03	00·83	06·95	06·90
24·39	39·63	2012/07/19	11·79	04	01·02	07·12	07·04
26·35	43·95	2012/07/19	11·56	03	00·68	07·06	07·03
25·62	45·62	2012/07/19	11·43	03	00·73	07·00	06·96
21·42	39·83	2012/09/16	13·20	05	01·34	02·85	02·52
24·60	46·45	2012/09/16	12·77	03	00·72	02·33	02·21
26·35	43·95	2012/09/16	12·94	02	00·53	02·26	02·19
25·25	45·25	2012/09/16	12·85	02	00·66	02·31	02·21
26·37	49·82	2012/09/16	12·55	01	00·39	02·06	02·02
25·62	45·62	2012/09/16	12·83	02	00·60	02·27	02·19
24·60	46·45	2012/11/14	15·99	33	06·70	08·81	05·73
26·35	43·95	2012/11/14	16·11	33	06·57	08·90	06·02
27·52	41·70	2012/11/14	16·23	33	06·49	08·98	06·22
28·40	36·73	2012/11/14	16·54	33	06·53	09·17	06·45
25·25	45·25	2012/11/14	16·06	33	06·66	08·85	05·84
25·62	45·62	2012/11/14	16·02	33	06·61	08·84	05·88
26·37	49·82	2012/11/14	15·72	32	06·38	08·67	05·88
21·42	39·83	2012/12/13	05·98	16	03·48	03·56	00·73
24·39	39·63	2012/12/13	05·89	16	03·47	03·50	00·51
24·60	46·45	2012/12/13	05·43	15	03·22	03·28	00·61
26·35	43·95	2012/12/13	05·54	16	03·30	03·33	00·44
27·52	41·70	2012/12/13	05·65	16	03·36	03·38	00·32
25·25	45·25	2012/12/13	05·49	16	03·26	03·30	00·54
25·62	45·62	2012/12/13	05·45	16	03·24	03·28	00·52
21·42	39·83	2013/02/10	07·93	16	03·78	06·09	04·78
24·39	39·63	2013/02/10	07·89	18	04·00	06·04	04·53
28·40	36·73	2013/02/10	08·00	20	04·38	06·03	04·15
25·25	45·25	2013/02/10	07·49	17	03·86	05·91	04·48

TABLE II (*concluded*)

Lat. °	*Long.* °	*Date* *UT*	*Age* *hrs.*	*Lag* *min.*	*ARCV* °	*ARCL* °	*DAZ* °
25·62	45·62	2013/02/10	07·46	17	03·87	05·90	04·45
26·37	49·82	2013/02/10	07·17	17	03·77	05·80	04·41
21·42	39·83	2013/04/10	06·05	09	02·09	02·95	02·08
25·62	45·62	2013/04/10	05·72	09	02·07	02·80	01·88
21·42	39·83	2013/10/05	02·25	15	03·43	07·28	06·43
24·39	39·63	2013/10/05	02·24	14	03·09	07·32	06·64
26·35	43·95	2013/10/05	01·94	12	02·74	07·20	06·66
27·52	41·70	2013/10/05	02·08	12	02·66	07·28	06·78
25·25	45·25	2013/10/05	01·86	12	02·84	07·15	06·56
25·62	45·62	2013/10/05	01·84	12	02·78	07·14	06·57
26·37	49·82	2013/10/05	01·55	12	02·59	07·01	06·52
21·42	39·83	2013/11/03	14·35	00	00·23	00·41	00·34
21·42	39·83	2014/01/01	17·19	12	02·64	04·58	03·74
24·39	39·63	2014/01/01	17·11	13	02·80	04·53	03·56
24·72	46·67	2014/01/01	16·63	12	02·56	04·44	03·63
26·35	43·95	2014/01/01	16·75	13	02·74	04·44	03·49
27·52	41·70	2014/01/01	16·86	14	02·87	04·44	03·39
28·40	36·73	2014/01/01	17·16	15	03·10	04·49	03·25
25·25	45·25	2014/01/01	16·70	12	02·63	04·44	03·58
25·62	45·62	2014/01/01	16·66	12	02·64	04·43	03·56
26·37	49·82	2014/01/01	16·36	12	02·52	04·38	03·58
21·42	39·83	2014/03/01	20·67	14	03·38	04·96	03·63
24·72	46·67	2014/03/01	20·18	14	03·30	04·74	03·41
26·35	43·95	2014/03/01	20·35	15	03·49	04·78	03·28
25·25	45·25	2014/03/01	20·27	15	03·38	04·77	03·37
25·62	45·62	2014/03/01	20·24	15	03·39	04·76	03·34
26·37	49·82	2014/03/01	19·96	14	03·27	04·65	03·31
21·42	39·83	2014/04/29	09·52	17	03·93	04·31	01·76
24·39	39·63	2014/04/29	09·59	18	03·86	04·36	02·03
28·40	36·73	2014/04/29	09·87	18	03·84	04·53	02·41
25·25	45·25	2014/04/29	09·24	17	03·66	04·19	02·06
25·62	45·62	2014/04/29	09·22	17	03·63	04·19	02·09
26·35	43·95	2014/07/27	17·21	08	01·81	08·91	08·72
21·42	39·83	2014/09/24	09·03	05	01·28	04·05	03·84
24·72	46·67	2014/09/24	08·57	04	00·90	03·91	03·81
26·35	43·95	2014/09/24	08·75	03	00·84	04·01	03·92
27·52	41·70	2014/09/24	08·90	03	00·82	04·08	04·00
28·40	36·73	2014/09/24	09·23	04	00·86	04·22	04·14
25·62	45·62	2014/09/24	08·64	03	00·86	03·95	03·86
26·35	43·95	2014/11/22	01·67	10	02·08	02·98	02·13
27·52	41·70	2014/11/22	01·78	10	02·17	02·97	02·03
21·42	39·83	2015/01/20	01·81	04	01·04	04·61	04·50
24·72	46·67	2015/01/20	01·26	04	01·01	04·57	04·46
26·35	43·95	2015/01/20	01·40	05	01·22	04·54	04·37
27·52	41·70	2015/01/20	01·51	06	01·38	04·52	04·31
28·40	36·73	2015/01/20	01·82	07	01·61	04·51	04·21
25·25	45·25	2015/01/20	01·34	04	01·10	04·56	04·43
25·62	45·62	2015/01/20	01·31	05	01·11	04·55	04·42
26·37	49·82	2015/01/20	01·00	04	01·00	04·55	04·44
21·42	39·83	2015/03/20	05·92	11	02·63	02·73	00·74
27·52	41·70	2015/03/20	05·80	11	02·62	02·65	00·38
28·40	36·73	2015/03/20	06·13	13	02·81	02·83	00·29
21·42	39·83	2015/05/18	11·67	21	04·68	07·04	05·27
24·39	39·63	2015/05/18	11·76	21	04·43	07·12	05·58
24·72	46·67	2015/05/18	11·30	19	04·16	06·92	05·54
26·35	43·95	2015/05/18	11·53	19	04·10	07·04	05·73
25·25	45·25	2015/05/18	11·41	19	04·16	06·98	05·61
25·62	45·62	2015/05/18	11·40	19	04·11	06·98	05·64
26·37	49·82	2015/05/18	11·14	18	03·90	06·87	05·66
21·42	39·83	2015/09/13	08·75	06	01·55	03·56	03·20
26·37	49·82	2015/09/13	08·11	04	01·06	03·36	03·19

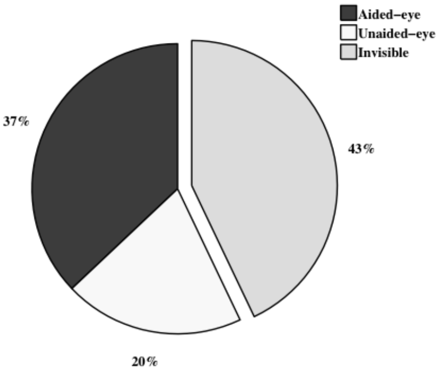

FIG. 3

Percentage of observations where the crescent was invisible (43%), and positive sightings made by optical means (telescope or binoculars, 37%), and by the naked-eye (20%).

TABLE III

Examples of observations and observational parameters

V/I refers to visible or invisible; T stands for telescope; and N stands for naked eye.

No.	*Lat.* °	*Long.* °	*Date* *UT*	*Age* *hrs.*	*Lag* *min.*	*ARCV* °	*ARCL* °	*DAZ* °	*W* ′	*V/I*
026	27·52	41·70	2000/12/26	21·08	40	07·70	08·86	04·40	0·19	V(T)
085	27·52	41·70	2009/01/27	30·92	64	13·35	13·55	02·32	0·44	V(N)
096	26·35	43·95	2009/10/19	33·00	35	06·99	17·47	16·05	0·77	V(T)
099	25·62	45·62	2009/11/18	42·90	70	12·92	20·51	16·07	1·01	V(N)
100	25·62	45·62	2009/12/17	26·13	48	09·08	11·50	07·07	0·32	V(N)
102	28·39	36·73	2010/04/15	27·49	65	13·20	13·51	02·85	0·44	V(N)
104	24·54	39·63	2010/06/13	28·94	68	13·87	15·22	06·34	0·56	V(N)
122	25·62	45·62	2010/12/06	20·93	40	07·87	10·95	06·49	0·25	V(T)
127	24·39	39·63	2011/04/04	25·00	50	10·70	11·42	04·04	0·31	V(N)
137	24·60	46·45	2011/09/28	27·60	32	06·98	16·55	15·04	0·66	V(T)
154	28·40	36·73	2011/12/25	20·61	54	10·30	10·78	03·18	0·53	V(N)
161	21·42	39·82	2012/04/22	32·00	64	13·83	13·87	01·06	0·47	V(N)
167	25·62	45·62	2012/07/20	35·42	43	09·50	17·80	15·13	0·76	V(N)
170	26·35	43·95	2012/09/17	36·92	45	09·64	20·34	18·00	0·99	V(N)
177	25·62	45·62	2012/10/16	26·42	40	08·28	14·88	12·14	0·53	V(N)
442	27·52	41·70	2012/11/14	16·23	33	06·49	08·98	06·22	0·19	I(-)
195	25·62	45·62	2013/01/12	18·71	53	10·87	10·93	01·25	0·29	V(N)
455	24·39	39·63	2013/02/10	07·88	18	04·00	06·04	04·53	0·08	I(-)
204	25·62	45·62	2013/04/11	29·72	63	13·23	13·56	00·05	0·45	V(N)
205	21·42	39·82	2013/05/10	15·37	28	06·07	06·34	01·85	0·10	I(-)
207	24·39	39·63	2013/10/06	38·50	59	12·46	19·65	15·33	0·93	V(N)
217	24·39	39·63	2013/12/03	14·18	39	07·92	08·10	01·62	0·16	V(T)
228	24·72	46·66	2014/01/31	16·98	46	09·86	10·31	02·99	0·25	V(N)
239	24·39	39·63	2014/03/31	20·87	48	10·47	10·40	00·30	0·27	V(N)
488	21·42	39·83	2014/04/29	09·51	18	03·93	04·30	01·76	0·05	I(-)
247	27·52	41·70	2014/05/29	21·37	36	07·33	10·47	07·49	0·27	V(T)
255	28·40	36·72	2014/09/25	33·20	41	08·65	15·07	12·39	0·55	V(N)
257	26·35	43·95	2014/11/23	25·66	62	12·30	13·41	05·36	0·43	V(N)
264	28·39	36·72	2015/02/19	15·65	41	08·72	08·82	01·30	0·19	V(N)
531	24·72	46·66	2015/05/18	11·30	19	04·15	06·92	05·54	0·12	I(-)

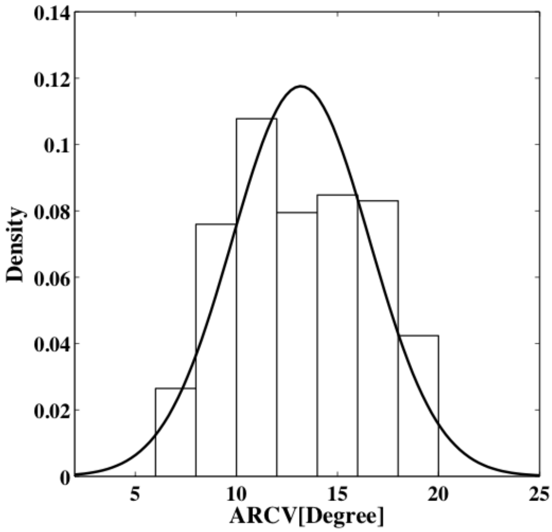

FIG. 4
Arc of vision (ARCV) distribution. The solid curve represents the best Gaussian fit of the distribution.

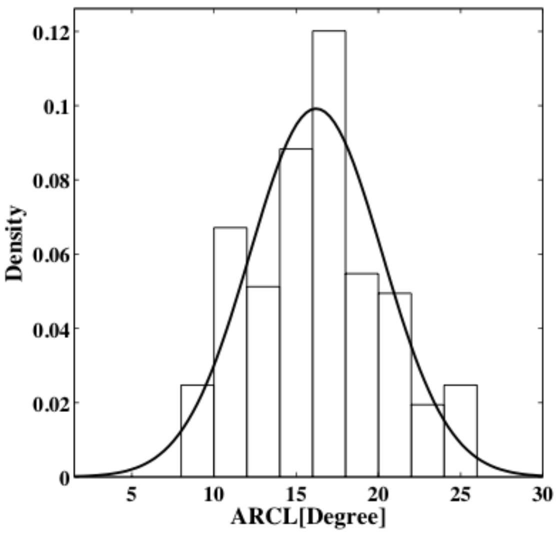

FIG. 5
Arc of light (ARCL) distribution. The solid curve represents the best Gaussian fit of the distribution.

Observation analysis

The data described in previous section and presented in Table I can be used to investigate the various prediction criteria for the first visibility of the lunar crescent following the new moon. We investigate the Moon's age and the lag time (LAG) as individual criteria. We examine the relationship between the Moon's age and lag time (LAG) with the two arcs (ARCV and ARCL). We study the relationships between the three arcs ARCV, ARCL, and DAZ and use these arcs to determine the best lunar-visibility criteria based on the collected data.

The Moon's age

The Moon's age is a simple parameter that has been used to predict lunar-crescent visibility. The youngest lunar crescent reported in our data set is 14·18 hrs, observed by telescope. The world record for the youngest visible lunar crescent by optical aid was 12·12 hrs on 1996 January 20[15]. The minimum Moon age in our data set, from observation 217 in Table III, has an ARCV value of 7°·92, and an ARCL value of 8°·10, which is the minimum ARCL in our data set. Observation 442 in Table III had a Moon age of 16·23 hrs, older than the minimum moon age of observation 217. The ARCV was 6°·49 and the ARCL was 8°·98, which is larger than observation 217; however, observation 442 was a negative sighting. A scatterplot of the age of the youngest visible lunar crescent *versus* ARCV and ARCL indicates a weak relationship between geometry arcs and the lunar-crescent age as shown in Figs. 6 and 7, respectively. These figures show a significant dispersion around the best linear fit and indicate that the minimum Moon age is a simple parameter but insufficient criterion for lunar visibility. The Moon takes a longer time to be visible after the new moon conjunction when it is closer to the ecliptic[6].

Moon lag (LAG)

The second most-simple and oldest lunar-visibility criterion is the Moon lag time (LAG). The minimum interval of time between the sunset and moonset (LAG) was 32 minutes (observation 137 in Table III), made by telescope, and the shortest interval ever recorded for a visible lunar crescent by optical aid was 21 min.[15]. As for Moon age, observation 442 had a longer Moon lag time (33 min.) but again, was a negative sighting. A scatterplot of LAG *versus* ARCV in Fig. 8 shows that a large number of observations have a linear relationship. There is some dispersion around the best-fit line, which indicates that a lunar crescent with a particular lag time may or may not be visible depending on the ARCV value.

We note that, the longer the LAG, the higher the altitude (ARCV) of the lunar crescent and the darker the sky. A scatterplot of LAG *versus* ARCL in Fig. 9 shows a significant dispersion around the best linear fit, which indicates a weak relationship between ARCL and LAG. Fig. 8 shows that LAG is a relatively applicable parameter.

The Danjon limit (ARCL)

The third direct criterion is the critical angular separation or elongation between the Moon and Sun at sunset or the arc of light (ARCL). This is very well known as the Danjon limit. According to the Danjon limit, the lunar crescent cannot be seen with the naked-eye if the angular separation between the Moon

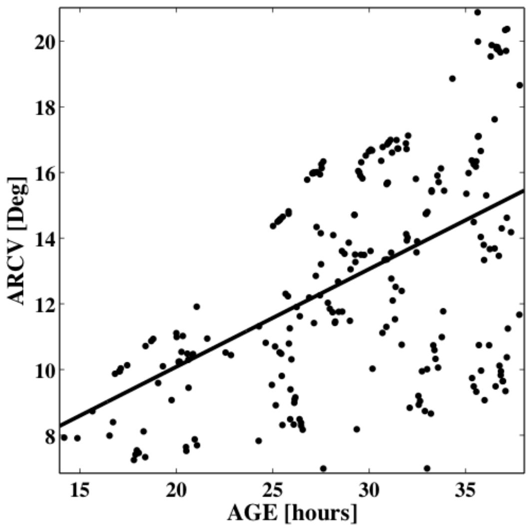

FIG. 6
Arc of vision (ARCV) *versus* lunar-crescent age. The solid line represents the best linear fit.

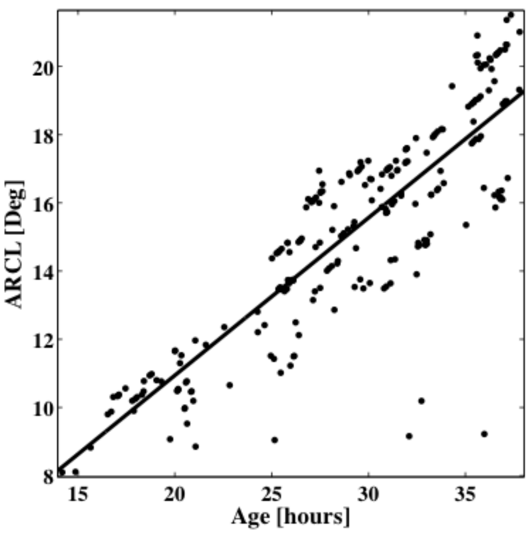

FIG. 7
Arc of light (ARCL) *versus* lunar-crescent age. The solid line represents the best linear fit.

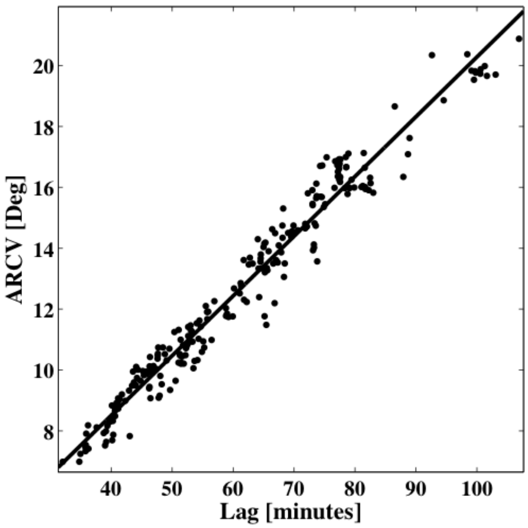

FIG. 8
Arc of vision (ARCV) *versus* (LAG). The solid line represents the best linear fit.

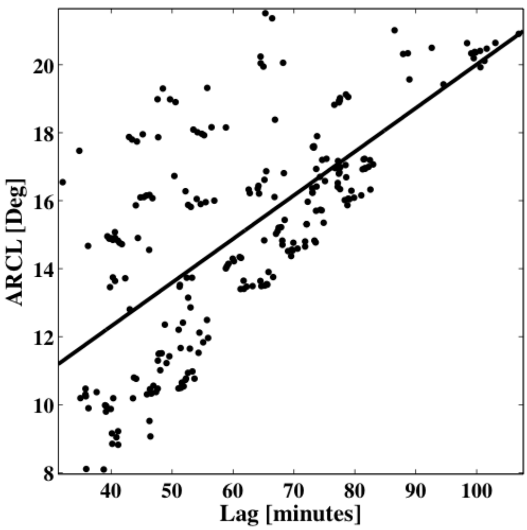

FIG. 9
Arc of vision (ARCL) *versus* (LAG). The solid line represents the best linear fit.

and Sun (ARCL) is less than 7°. Equation (1) relates the two independent arcs ARCV and DAZ with the arc of light (ARCL):

$$\cos(\text{ARCL}) = \cos(\text{ARCV})\cos(\text{DAZ}). \qquad (1)$$

where all the arcs are measured in degrees. The Doggett & Schaefer[1] lunar-crescent-sighting campaign reinforced the Danjon limit. In our data set, the minimum recorded ARCL value by naked-eye was 8°·82 in observation 268 (Table III), which is larger than the Danjon limit. Even by aided-eye, the minimum recorded ARCL was 8°·10 in observation 217, which is larger than the Danjon limit. Ilyas[3,16] determined the lower limit of ARCL observed by naked-eye as 10°·5. Our data set contains 27 observations with ARCL < 10°·5, indicating that Ilyas's criterion is certainly overestimated (perhaps due to data collected with non-computerized mounted telescopes). In 2004, Odeh[15] claimed that he found one observation where the lunar crescent was seen for ARCL = 6°·40. Observations that exceed the Danjon limit are rare and care should be taken to investigate their validity.

The visibility curve (DAZ–ARCV)

The dividing (DAZ–ARCV) curve or lunar-visibility curve is used to separate regions of positive and negative lunar-crescent sightings based on the two arcs DAZ and ARCV, illustrated in Fig. 2. In 1910, Fotheringham used a collection of 76 lunar-crescent sightings, most observed in 1883 in Athens and Troy[7], to calculate the arc of vision (ARCV) and relative azimuth (DAZ). He found that the crescent was visible only when ARCV was above a critical value that depends on DAZ. The formula for lunar-crescent visibility is given by

$$\text{ARCV} = 12 - 0{\cdot}008(\text{DAZ}^2), \qquad (2)$$

where ARCV is the minimum Moon altitude at sunset in degrees and DAZ is the difference in azimuth in degrees[7]. Equation (2) is shown in Fig. 2, where the solid curve is the dividing line between visible (above the curve) and invisible (below the curve) lunar crescents. In 1911, Maunder added 47 observations to the collection and only used positive sightings and the quadratic equation to represent the data[8]. The Maunder criterion for lunar-crescent visibility is given by

$$\text{ARCV} > 11 - 0{\cdot}05(\text{DAZ}) - 0{\cdot}01(\text{DAZ}^2), \qquad (3)$$

where ARCV and DAZ are in degrees. The dot-dash curve in Fig. 2 reveals that the Maunder dividing curve is lower than that of Fotheringham[8]. The Maunder criterion (equation (3)) is slightly modified in the *Indian Astronomical Ephemeris*. The Indian formula is based on the Maunder criterion, which fits DAZ to ARCV and produces the following equation:

$$\text{ARCV} > 10{\cdot}3743 - 0{\cdot}0137(\text{DAZ}) - 0{\cdot}0097(\text{DAZ}^2), \qquad (4)$$

where ARCV and DAZ are in degrees[3,12]. This formula is used in the *Indian Astronomical Ephemeris*[9] and shown in Fig. 2 as the dashed curve, which indicates that the Indian criterion is slightly different to the Maunder criterion.

Our data set was used to design a crescent-visibility curve that separates visible and non-visible lunar-crescent regions characterized by the two arcs ARCV and DAZ. This method follows that of Fotheringham[7]. The data set is divided into two parts: one for observations by naked-eye; and one for observations by aided eye. For unaided observations, a scatterplot is produced in the ARCV–DAZ plane (Fig. 10), where lunar-crescent observations are divided into two clear regions. The crosses in Fig. 10 represent all invisible observations and the solid

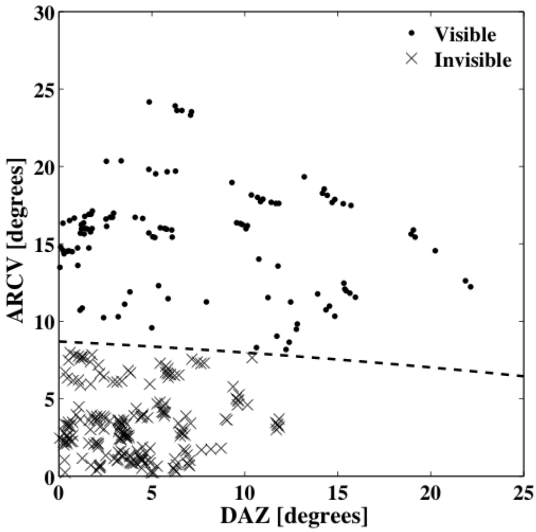

FIG. 10

Hypothetical curve dividing unaided-eye lunar-crescent observations into two regions: visible (above the curve) and invisible (below the curve).

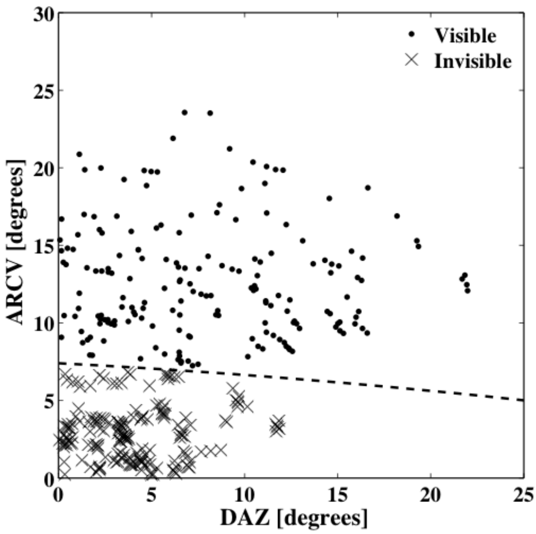

FIG. 11

Hypothetical curve dividing aided-eye lunar-crescent observations into two regions: visible (above the curve) and invisible (below the curve).

points represent all visible observations by naked-eye. A visual inspection of that figure led to the insertion of a hypothetical line separating the data into two regions of points representing visible and invisible lunar-crescent observations. The best hypothetical curve is given by the formula

$$\mathrm{ARCV} = 8{\cdot}7 - 0{\cdot}0594(\mathrm{DAZ}) - 0{\cdot}00121(\mathrm{DAZ}^2), \qquad (5)$$

where ARCV and DAZ are in degrees. Similarly, we perform the same procedure for aided-eye observations (Fig. 11). The hypothetical curve in that figure is given by the formula

$$\mathrm{ARCV} = 7{\cdot}40 - 0{\cdot}0624(\mathrm{DAZ}) - 0{\cdot}0013(\mathrm{DAZ}^2). \qquad (6)$$

Equations (5) and (6) are based only on visual inspection using the best hypothetical curve. The ARCV–DAZ dividing curve reveals that the critical arc of vision (ARCV) of the thin crescent after sunset can be small when the azimuthal distance (DAZ) between the Moon and Sun is large. Thus, for the lunar crescent to be visible, the western sky should be sufficiently dark. Positive (negative) lunar-crescent sightings above (below) the (ARCV–DAZ) dividing curve depend mainly on the contrast between sky-brightness and thin-lunar-crescent illumination. A small contrast can be the difference between visible and non-visible sightings. The contrast increases to the width of the lunar-crescent (W), which is related to the arc of light (ARCL), and therefore to the arc of vision (ARCV) and the azimuthal distance (DAZ) (see equation (1)), according to the following formula

$$W = SD(1 - \cos(\mathrm{ARCL})), \qquad (7)$$

where SD is the semi-diameter of the Moon, which is related to the Moon's parallax π by $SD = 0{\cdot}27255\pi$. In this study, we adopted an SD value of 16′. For the western sky, the contrast is determined by ARCV and W. The minimum contrast resulting in a visible lunar crescent depends on the direct relationship between ARCV and W. Using equation (5) for the best hypothetical curve for naked-eye observations, for any given DAZ, the ARCV can be obtained. The critical contrast between visible and not visible is deduced in Table IV using equations (5), (1), and (7). In Fig. 12, a third-degree polynomial is fitted to the data in Table IV. The resulting critical ARCV required for the critical contrast for naked-eye lunar-crescent sightings is shown by equation (8):

$$\mathrm{ARCV} > 9{\cdot}34 - 4{\cdot}51W + 3{\cdot}3W^2 - 1{\cdot}01W^3. \qquad (8)$$

TABLE IV

ARCV and W for certain values of DAZ by using Equation (5)

DAZ (°)	0	5	10	15	20	25
ARCV (°)	8·70	8·37	7·89	7·53	7·02	6·45
W (′)	0·18	0·23	0·39	0·67	1·07	1·59

The same procedure is performed for aided lunar-crescent sightings, using the hypothetical line equation for aided lunar-crescent observations (equation (6)). ARCV is calculated for certain values of DAZ shown in Table V. The critical W for visibility is deduced in that table using equations (6), (1), and (7). In Fig. 13, a third-degree polynomial is fitted to the data given in Table V. The resulting critical ARCV required for the critical contrast for aided-eye lunar-crescent sightings is shown by equation (9):

$$\mathrm{ARCV} > 7{\cdot}83 - 4{\cdot}35W + 3{\cdot}22W^2 - 1{\cdot}02W^3. \qquad (9)$$

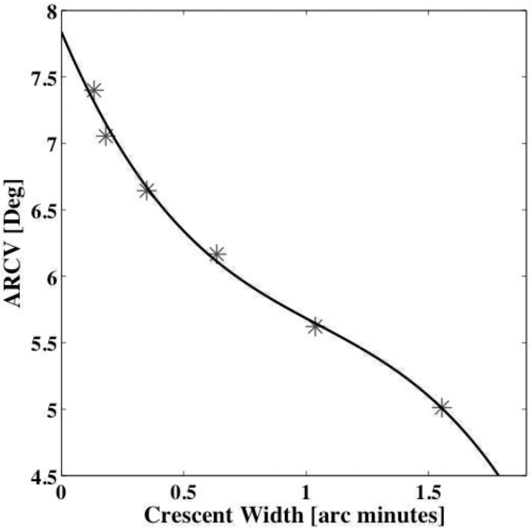

FIG. 12
Third-degree polynomial fitted to the data given in Table IV.

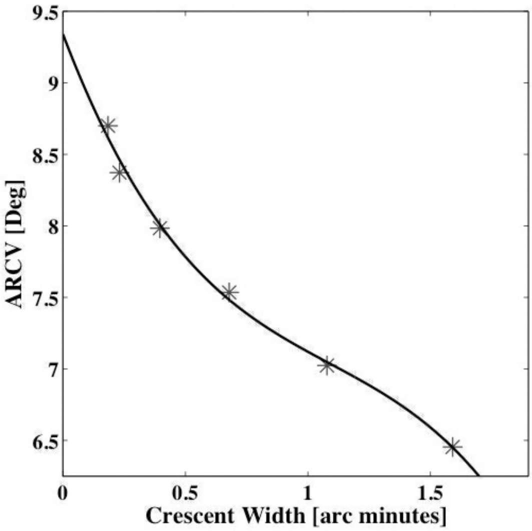

FIG. 13
Third-degree polynomial fitted to the data given in Table V.

TABLE V

ARCV and W for certain values of DAZ by using Equation (6)

DAZ (°)	0	5	10	15	20	25
ARCV (°)	7·40	7·05	6·64	6·16	5·62	5·01
W (′)	0·13	0·18	0·34	0·63	1·03	1·55

The new critical ARCV values in equations (8) and (9) were determined using the visibility curve (DAZ–ARCV). After a visual inspection of the naked-eye and aided-eye scatterplots (Figs. 10 & 11), a hypothetical line ARCV = *f*(DAZ) was inserted, separating regions of positive and negative sightings of the youngest lunar crescent. Equations for these hypothetical lines were used to deduce the numerical values for ARCV and *W* (Tables (IV & V)). These values were then fitted using a third-degree polynomial in the lunar-crescent width (*W*) equations (8, 9).

Discussion

We used a large set of observations of the youngest lunar crescent to investigate several criteria for visibility of the first thin lunar crescent following a new-moon conjunction. We first analyzed Moon age as an individual criterion that has been used since the Babylonians[3]. The youngest lunar crescent observed by telescope in our data set was 14·18 hrs. Our minimum Moon age is older than found by Schaefer *et al.*[17] during their 1987–1990 campaign (13·47 hrs), and older than the world record (12·12 hrs)[15]. It is clear, therefore, that the youngest Moon age after the lunar–solar conjunction is not sufficient for determining thin-lunar-crescent visibility (Figs. 6 & 7). When the Moon is near the ecliptic, it takes longer to increase its brightness and become visible. The second criterion we investigated was Moon lag time (LAG). We found that the Moon lag time is not related to the arc of light (ARCL) (Fig. 9) and that observations with a larger LAG include negative sightings, such as observation 442 in Table III. Fig. 9 shows that LAG does not contribute to greater contrast.

The third criterion we investigated was the Danjon limit, which states[11] that the arc of light (ARCL) required for the lunar crescent to be visible must be greater than 7°. However, a re-examination of Danjon's data by Ilyas[10] showed that, if one point below 10° were removed from Danjon's data, the limit increases to 10°·5. McNally[18] explained that the Danjon limit was due to atmospheric effects or turbulence, and found that the angular size of the lunar crescent vanishes for arc of light (ARCL) values of ≈ 5°. This explanation was rejected by Schaefer[19], who proposed that the lunar cusps are not visible because the brightness per unit length is below the eye's detection threshold, and who concluded that the Danjon limit is quite solid. Fatoohi *et al.*[20] reported an increase in the Danjon limit to 7°·5, and the 7° suggested by Danjon[11] and accepted by Scheafer[19] appears to overestimate the 10°·5 limit of Ilyas[10].

We found that lunar crescents within the Danjon limit at ARCL ≤ 7° were not detected by naked-eye or aided-eye observations. In our data set, the minimum ARCL observed by naked-eye was 8°·82 and that observed by aided-eye was 8°·10. Our results agree with those of Fatoohi *et al.*[20]. After investigating the main parameters commonly used as criteria for early lunar-crescent visibility, we followed the procedure used by Fotheringham[7] and others to determine the critical visibility curve (Figs. 10 & 11). We found new criteria for early new-moon visibility by naked-eye and aided-eye observations (equations (8, 9)). This method using the DAZ–ARCV curve was also used by Yallop[12], who used the Indian formula given in the *Indian Astronomical Ephemeris*[9], whereby numerical values for the arc of vision and the lunar-crescent width (ARCV, W) are fitted using a third-degree polynomial. Then Yallop developed his two-parameter criterion into a single-parameter criterion for visibility of the lunar crescent, in which the q-values are defined as

$$q = (\mathrm{ARCV} - (11{\cdot}871 - 6{\cdot}3226W + 0{\cdot}7319W^2 - 0{\cdot}1018W^3))/10. \qquad (10)$$

The division by 10 in equation (10) confines[12] the values of q between –1 and 1. When q-values are $-0{\cdot}232 \geq q > -0{\cdot}293$, the lunar crescent is not visible with a telescope[12].

We investigated Yallop's criterion by applying it to our data collection. Yallop's criterion (equation (10)) is easy to convert to arc of vision (ARCV) in terms of the azimuthal distance (DAZ). We then used our aided-eye lunar-crescent observations to produce a scatterplot in the ARCV–DAZ as shown in Fig. 14, where the solid curve represents the Yallop criterion and the invisible region is below the solid curve. Clearly, Yallop's criterion lies above some observations from our data collection, and uses semi-topocentric values for the lunar-crescent width (W)[12]. We investigated his criterion using our data of both semi-topocentric (W') and geocentric lunar-crescent width (W) as shown in Fig. 15. There is little difference between the two values of the lunar-crescent width. The dashed line in Fig. 15 is the q-value ($q = -0{\cdot}232$) in Yallop's criterion, which is not visible by telescope. We found some of our observations by aided-eye lie below that line, in the invisible region, based on Yallop's criterion (Table VI). From Figs. 14 & 15, Yallop's criterion is not consistent with our data for lunar-crescent observations, and does not consider how the semi-topocentric W improved the fit to the observations[12]. In 2004 Odeh continued the work of Yallop and found a new criterion based on ARCV–DAZ curve and used 737 observations[15]. Odeh fitted numerical values using a third-degree polynomial, which provides the following criterion:

$$V = \mathrm{ARCV} - (7{\cdot}1651 - 6{\cdot}3226W + 0{\cdot}7319W^2 - 0{\cdot}1018W^3). \qquad (11)$$

When we compare Odeh's model (equation (11)) with that of Yallop (equation (10)), Odeh's numerical values of the coefficient are similar up to the fifth digit to the Yallop's model, but Yallop's division by 10 confines the values of q between –1 and 1. We investigated Odeh's criterion based on the ARCV–DAZ visibility curve, and found that the dashed curve in Fig. 14 is below some observations classified as invisible in our data set. According to the ARCV–DAZ plane (Fig. 14), Odeh's is the same as Yallop's but shifted down. This similarity is questionable because both used different data sets of early lunar-crescent observations. Odeh's used topocentric values for the lunar-crescent width (W) and arc of vision (ARCV), but did not discuss how the topocentric variables improve the criterion. Moreover, we found that the third-degree polynomial fit for lunar-crescent width (W) with the arc of vision (ARCV) given in Table V, page 43, of ref. 20 is not consistent with his model (equation (11)).

Therefore, we propose that the criterion presented in this study for early crescent visibility is the latest criterion based on accurate observations taken by highly trained astronomers. Our two models (equations (8 and 9)) are thoroughly described and suitable for determining the lunar-crescent calendar

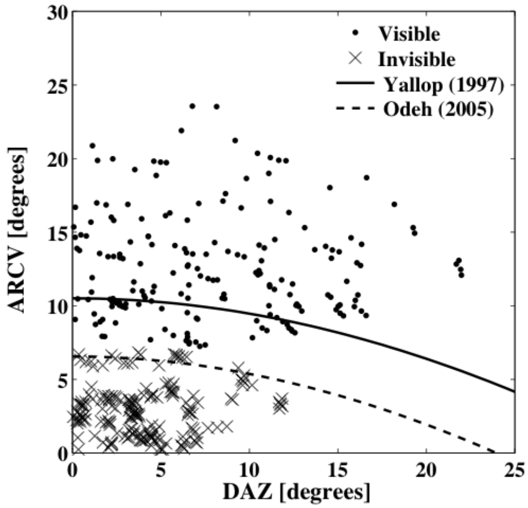

FIG. 14
Yallop and Odeh models in the ARCV–DAZ plane with our eye-aided observation data.

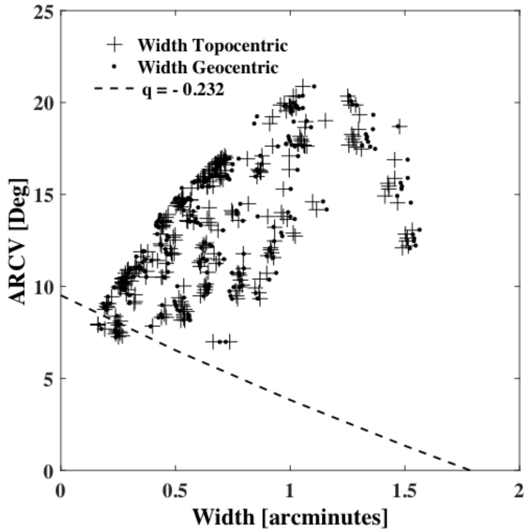

FIG. 15
Application of the Yallop criterion to the geocentric and semi-topocentric lunar-crescent width.

TABLE VI

Observations not in agreement with Yallop's criterion

No.	*Lat.* °	*Long.* °	*Date* *UT*	*ARCV* °	*ARCL* °	*DAZ* °	*W* ′	*q*
026	27·52	41·70	2000/12/26	07·70	08·86	04·40	00·18	−0·31
038	27·52	41·70	2002/11/05	07·41	09·90	06·58	00·25	−0·29
064	27·52	41·70	2005/02/09	07·98	09·80	05·70	00·24	−0·24
075	24·54	39·63	2006/02/28	07·91	08·12	01·81	00·17	−0·29
115	24·39	39·63	2010/12/06	07·87	10·20	06·50	00·25	−0·24
116	24·60	46·45	2010/12/06	07·63	09·97	06·42	00·24	−0·27
117	25·62	45·62	2010/12/06	07·52	09·99	06·60	00·24	−0·28
217	24·39	39·63	2013/12/03	07·92	08·10	01·62	00·17	−0·29
246	24·72	46·67	2014/05/29	07·54	10·25	06·97	00·24	−0·28
247	27·52	41·70	2014/05/29	07·33	10·47	07·49	00·25	−0·30
248	25·62	45·62	2014/05/29	07·46	10·31	07·13	00·24	−0·29
249	26·37	49·82	2014/05/29	07·24	10·20	07·20	00·24	−0·31

for civil purposes. The criteria involve both naked-eye and aided-eye sightings; however, they are not applicable on a global scale because our observations were made at latitudes between 20° N and 29° N.

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